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ROYAL AIRCRAFT ESTABLISHMENT.

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Note on dimensional relationships for air compressors.

-by-

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Summary.

R. Dimensional relationships are derived by which the performance of a compressor may be deduced at one set of intake conditions from tests at another. It is shown that the dimensional relationships are unaltered by heat transfers across the walls of the compressor provided that the external cooling (or heating) arrangements are not varied, a condition met with for example in cooling by an airstream of fixed speed, density, and temperature.

Assuming the scale effect variable to have a negligible effect, it is shown that the compression ratio, temperature ratio, and efficiency are unaltered if:-

- (a) the mass flow varies as the geometric mean of the pressure and density of the air at the intake,
- (b) the tip speed varies as the square root of the ratio of the intake pressure and density.

For constant intake conditions and constant tip speed of the rotor, the output varies as the square of the linear dimensions.

Tests are suggested to check the assumption that scale effect may be neglected and to discover the effect of heating the casing in order to ascertain if the high temperatures which occur in the casings of exhaust driven centrifugal compressors have a detrimental effect on performance.

1. Dimensional relationships when walls of compressor are non-conducting.

The physical quantities associated with the performance of a compressor, whether of the centrifugal or displacement type, are

P_0 and P_1 , the pressures at intake and delivery.

ρ_0 and ρ_1 , the densities " " " "

T_0 and T_1 , the temperatures " " "

P the power required to operate the compressor.

W the mass of air dealt with in unit time, i.e. the mass flow.

ν the kinematic viscosity.

D a linear dimension.

n the rate of rotation of the rotor.

The frictional losses in the bearings will be neglected (or alternatively P may be assumed to be the power required less such losses), and it will further be supposed in the first instance, that there are no heat transferences across the walls.

Only six of the above variables are independent: for example, if n , D , P_0 , ρ_0 , W and ν are given, P , P_1 , and ρ_1 , may be derived. Alternatively P_1 may be considered to be an independent variable, W then becoming dependent.

By the customary procedure for deriving relationships based on a consideration of the dimensions of the quantities

involved we find that any non-dimensional quantity associated with the performance of the compressor may be expressed as a function of three non-dimensional variables built up from the six variables chosen as independent. In this way we find that

$\frac{p_1}{p_0}, \frac{\rho_1}{\rho_0}, \frac{T_1}{T_0}, \frac{P}{W n^2 D^2}$ and the adiabatic efficiency are functions of

$$\frac{W}{D^2 \sqrt{p_0} \rho_0} \quad n D \sqrt{\frac{\rho_0}{p_0}} \quad \frac{n D^2}{\sqrt{\gamma}}$$

The form of the functional relationships will be the same for compressors which are geometrically similar. If, therefore, the effect of the variable $\frac{n D^2}{\sqrt{\gamma}}$ is small, it follows that for given intake conditions and for a given $n D$ (i.e. for a given impeller tip speed in a centrifugal compressor) the efficiency, compression ratio, temperature rise through the compressor, and the ratio of power required to mass flow will be the same for mass flows proportional to the square of the linear dimensions, in compressors of similar geometrical form.

The way in which the performance of the compressor varies with atmospheric conditions is at once evident. In the standard atmosphere, p_0/ρ_0 varies only to a small extent, hence for a given impeller speed and given compression ratio,

efficiency, etc. the mass flow varies approximately as $\sqrt{P_0 \rho_0}$. By plotting compression ratio, efficiency etc. against $W/D^2 \sqrt{P_0 \rho_0}$ for constant values of $nD \sqrt{\frac{P_0}{\rho_0}}$ a set of curves would be obtained which would be applicable to all intake conditions, provided the effect of the variable nD^2/ν were small.

2. The effect of including heat transferences through the walls of the compressor casing.

It will be shown that no new non-dimensional variables are required if the cooling (or heating) conditions are invariant. This is most conveniently illustrated by an example. Let us suppose the compressor casing to be cooled (or heated) by a stream of air whose speed, pressure, and density at infinity are V , P_a , and ρ_a respectively. Then if $\frac{dH}{dt}$ is the rate of cooling per unit area at the point x, y, z of the casing where the temperature is θ ,

$$\frac{dH}{dt} = f(V, P_a, \rho_a, \nu, D, \theta, x, y, z)^*$$

The rate of communication of heat to the case by the working fluid at x, y, z is given by

$$\frac{dh}{dt} = f(p, \rho, \nu, D, W, n, \theta, x, y, z)$$

* Note: It is customary in discussing dimensional problems associated with the physical changes in a gas, where the temperature is explicitly referred to, to include non-dimensional quantities depending on the conductivity k , the viscosity μ , and the specific heats, e.g. such quantities as $\mu C_p/k$. All such quantities have a constant value: when therefore functional relationships between variables only are required, as in the present case, they need not be included explicitly.

If the conditions are steady $\frac{dH}{dt} + \frac{dh}{dt} = 0$, giving an equation connecting θ , x , y , z , the 'cooling conditions' V , p_a , p_a , and the six variables taken as independent in para.1. If therefore the 'cooling conditions' are constant,

θ may be replaced by the six independent variables, and no new variable is introduced, though the form of dependence of the dependent on the independent variables is, of course, altered. The performance of the compressor may therefore be plotted in a diagram of form similar to that suggested at the end of para.1, and the laws of variation of performance with intake conditions are unaltered.

It is easily seen that the above conclusions are not dependent on the form of cooling (or heating) provided that it may be reduced to dependence on quantities which do not vary.

If the quantities on which the cooling depends vary, then it may be shown that if θ_c is the temperature at a point on the compressor casing, the variation may be taken into account under certain circumstances by the introduction of a variable such as $\frac{nU}{\sqrt{\theta_c c_p}}$. No practical use for such a variable appears to arise, however.

3. Suggestions for test.

Bench tests of compressors appear to be required:-

- (a) to verify that the compression ratio, efficiency, and temperature ratio, when plotted against $W/D^2 \sqrt{p_a p_o}$

give unique families of curves with parameter $nD\sqrt{\frac{\rho_o}{p_o}}$ for ranges of nD , p_o , and ρ_o such as occur in practice.

(b) to ascertain the effect on such a set of curves of heating (or cooling) the compressor casing.

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